

Bearing Capacity of Steel-Concrete Plates

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Abstract— The preoccupation with an improvement of constructions in civil engineering leads to the adoption of the more powerful systems according to resistance, safety, the technology of implementation and also to the cost. The calculation of the steel-concrete elements was based beforehand on the experimental established formulae. This article proposes theoretical bases for the calculation of the interior forces, the normal and tangential constraints in the element as well as the acceptable load estimating in the steel-concrete plates. These formulae are justified by experimental data obtained. The divergence between these experimental values and the theoretical results obtained does not exceed 13,9%.

Keywords— Anchors, Bearing capacity, Steel-concrete plate, Ultimate load, Ultimate moment.

I. INTRODUCTION

The problem of development of theory and methods for calculation of structures with external reinforcement is attracting more and more attention (Voronkov, 1975 and Skorobogatov et al, 1985). This is caused by the practical demands which require application of these structures in order to obtain higher strength and rigidity of low-height floors (Voronkov, 1975), as well as to reduce metal intensity use, costs and labour input in construction.

Let us examine a hinged-supported steel-concrete plate the surface of which experiences uniformly distributed loading. Cross bonds which prevent steel sheet peeling from the concrete, are considered to be absolutely rigid both along the supporting contour, and in the span. Basic assumptions of thin plate's theory are considered to hold true. Integration of the sheet with concrete is effected with cylindrical anchors located symmetrically at equal intervals. The width (intervals) between anchor supports Δ is defined from the conditions of equality of ultimate loads along the contact and normal cross-section.

II. BREAKING LOAD ALONG THE CONTACT

A. The ultimate load on the anchor

The ultimate load on the anchor is defined from the condition of concrete strain or anchor cut according to the well-known relationships (Streletsky, 1981):

$$Q_a = 316 \cdot d_{anch}^2 (R_b)^{0.5}, (1)$$

$$Q_a = 6.3 \cdot d_{anch}^2 \cdot R_{sw}, (2)$$

Where R_{sw} is a design resistance anchor strain (MPa); R_b is design concrete compression strength (MPa); d_{anch} is anchor diameter (cm). While anchors are uniformly located across the contact area, the anchor supports situated along the supporting contour experience most of loading. It is evident, that ultimate state in these anchors will occur sooner than in the other ones. This gives ground to suggest a kinematic scheme of ultimate state which is shown in figure 1a. Cross-hatched sections in the scheme correspond to contact creeping zones, while the medium part of the plate (rectangular A'B'C'D') remains horizontal.

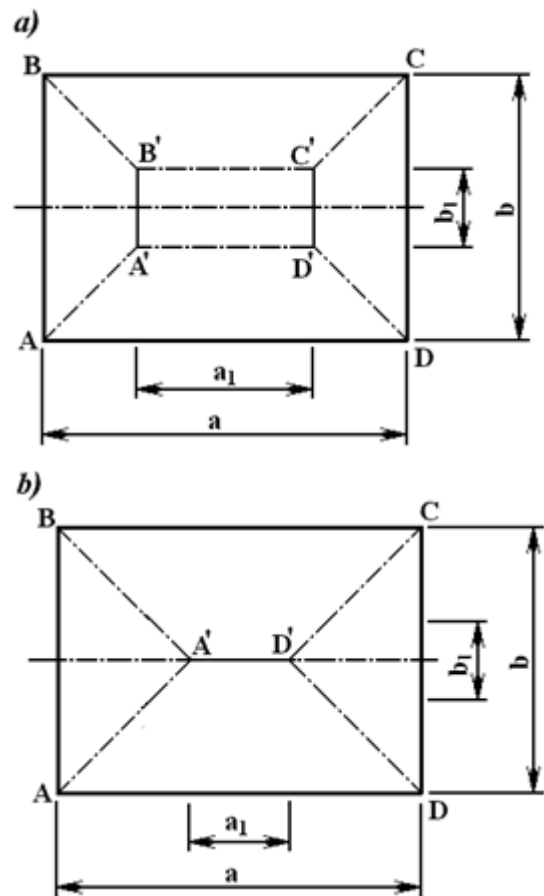


Figure 1. Scheme of Plates Flexure: a) Scheme of destruction with a rectangular medium part b) Envelop scheme of destruction

B. Determination of anchors

Taking into consideration one prerequisites, according to which flexures are insignificant in comparison with the plate height, let us find the angles of disks mutual turning during unit vertical displacement:

$$\begin{cases} \alpha_1 = 2/(b-b_1) \\ \alpha_2 = 2/(a-a_1) \\ \alpha_3 = \alpha_1 \cos \varphi + \alpha_2 \sin \varphi \end{cases} \quad (3)$$

The equation of the work of the system's external and internal forces at infinitely small displacements has the following form:

$$V = A = A_{anch} + A_{flex} \quad (4)$$

Work of external forces:

$$V = q[2(ab + a_1b_1) + ab_1 + a_1b]/6 \quad (5)$$

Work of internal forces is a combination of work of anchors A_{anch} and work of efforts emerging in the cross-section in the result of flexure A_{flex} . Anchors' work is expressed by the dependence:

$$A_{anch} = 2\tau_a (\Gamma_1 S_1 + \Gamma_2 S_2), \quad (6)$$

Where $\tau_a = Q_a / \Delta^2$; Γ_1 and Γ_2 present shift along the contact in the limits of disks ABA'B' and BCC'B' respectively, $\Gamma_1 = \alpha_1(h_b - x)$; $\Gamma_2 = \alpha_2(h_b - x)$; h_b is concrete layer height; x is concrete compressed zone height; thus, it follows from simple geometric considerations that $x = \varepsilon_1 / \alpha_1 = \varepsilon_2 / \alpha_2 = \varepsilon_3 / \alpha_3$; S_1, S_2 are corresponding disks areas, $S_1 = (a + a_1)(b - b_1)/4$, $S_2 = (b + b_1)(a - a_1)/4$. The work of efforts emerging in cross-section as the result of flexure looks like this:

$$A_{flex} = \sum_{i=1}^8 M_i \alpha_i l_i, \quad (7)$$

Where l_i is a length of i -th flexure section; M_i is running moment along the i -th flexure section; $M_i = R_{bi} x^2 / 2$; α_i is the turning angle of adjacent disks.

On substituting (5) – (7) into (4) and adopting $R_{bi} = R_{bi}$, as numerical calculations showed, it may result in error, the order of which does not exceed 1%, after simple transformations we obtain an equation for breaking load definition:

$$q_p = \frac{6\tau_a (h_b - 0.5x)(a + b + a_1 + b_1)}{[2(ab + a_1b_1) + ab_1 + a_1b]} \quad (8)$$

Where $x = \tau_a (a + a_1)(b - b_1) / (4aR_b)$.

Using condition $S_1/a = S_2/b$, we express a_1 through a, b, b_1 :

$$a_1 = \frac{[(b + b_1)a^2 + (b_1 - b)ab]}{(b^2 - bb_1 + b_1a + ba)} \quad (9)$$

Numerical analysis of equation (8) showed that to obtain minimum breaking load $b_1 = b - 2\Delta$ should be adopted.

III. BREAKING LOAD ACROSS NORMAL CROSS-SECTION

A. The ultimate moment

Let us assume as in (Gvozdev, 1949), that plate destruction will occur according to the known scheme of "envelope" (figure 1b), in this case the value of $\alpha_i = \sigma_2 / \sigma_1$ in concrete and steel is constant, or, at least, is varying insignificantly with occurring of non-linear deformations in the structure. Then the value of α_i , found from the elasticity calculation, will hold true for the points of the plate along the line of plastic hinge in the ultimate state.

The ultimate moment along the length of plastic hinge is defined in the following way:

$$M_i = \int_0^1 A_s \sigma_{si} (h_0 - 0.5A_s \sigma_{si} / R_{bi}) dx, \quad (10)$$

Where A_s is area of sheet reinforcement per unit of plastic hinge length; σ_{si}, R_{bi} are ultimate stresses in the steel sheet and concrete, with biaxial stressed state being taken into consideration. The criterion of creeping in the steel sheet is the condition of plasticity in conformity with Mises energetic theory. To define R_{bi} , the hypothesis of concrete strength is used (Kudzis and Notkus, 1977).

B. The ultimate load

As earlier, on making the equations which characterise work of external and internal forces and balancing them, we obtain an expression for definition of ultimate load during plate breaking across the normal cross-section:

$$q_p = \frac{24[M_1 \operatorname{ctg} \varphi + (M_1 + M_2) \operatorname{ctg} \varphi + aM_2/b]}{(ab(3-\nu))} \quad (11)$$

$$\text{Where } M_1 = \sum_{i=1}^n M_i / n, M_2 = \sum_{j=1}^m M_j / m,$$

$$\operatorname{ctg} \varphi = b/(va), \operatorname{ctg} \nu = (va)/b,$$

n, m are respectively the number of sections into which the line AA' and A'D' of plastic hinge is divided. The results of numerical calculations at computer help to find such value of φ at which the value of breaking load will be minimal.

IV. THE ANALYSIS OF NUMERICAL CALCULATIONS

To test the suggested dependences, the known reinforced-concrete plates of Bach Ts. and Graph O. and steel-concrete plates which are equivalent to them in terms of working reinforcement consumption, were calculated (table 1). During calculation of reinforced-concrete plates, Mises condition of plasticity was excluded, and stresses in reinforcement were adopted to be equal to σ_T . Steel-concrete plates were calculated with the same characteristics (h_0, R_b, σ_T) , as the reinforced-concrete ones. In table 1 q_T, q_T^* mean theoretical values of ultimate loads respectively for reinforced-concrete and steel-concrete plates proceeding from the condition of strength across the normal cross-section; q_{exp} is an experimental value of the ultimate load.

TABLE 1
Divergence of experimental and theoretical values of loading in Reinforced and Steel-Concrete plates

№ of plate, dimensions, m	Reinforced-concrete plates						Steel-concrete plates			
	$\mu_{ax} + \mu_{ay}$, cm ²	h_0 , cm	R_b , MPa	σ_T , MPa	q_T , t/m ²	q_{exp} , t/m ²	$\frac{q_{\text{exp}} - q_T}{q_T}$, %	δ , cm	q_T^* , t/m ²	$\frac{q_T^* - q_T}{q_T}$, %
P22, (2×2)	0.0814	6.35	26.5	408.0	6.03	6.3	4.2	0.0814	10.65	76.5
P32, (3×2)	0.077	10.5	26.5	427.9	7.17	7.45	3.7	0.077	13.61	89.8

Calculation of steel-concrete beams was also effected (Golosov et al., 1977), table 2, the destruction occurred along the contact. Divergence of experimental F_{exp} and theoretical values of loading along the contact F_T^c does not exceed 13.9%. As it was expected, ultimate loads across the normal cross-section F_T^n turned out to be

considerably higher than the experimental ones and those theoretically found.

This can be explained by the fact that interval between anchoring rods was adopted more than required. The required values of interval u_p are defined from the condition of equality of ultimate loads across the normal cross-section and along the contact and are presented in table 2 (values of interval between anchors adopted in the samples, are given in brackets).

TABLE 2
Divergence of experimental and theoretical values of Ultimate Loads Across the Normal Cross-Section and Along the Contact

Sample №	R_k , MPa	σ_T , MPa	F_T^c , t	F_T^n , t	F_{exp} , t	$\frac{F_{\text{exp}} - F_T^c}{F_T^c}$, %	u_p , cm
B-1-1a	54.8	400.0	12.3	38.9	11.1	-9.7	4.7 (15.0)
B-1-1b	54.8	400.0	18.0	40.4	20.5	13.9	4.5 (10.0)
B-2-1a	54.8	400.0	18.8	39.7	17.3	-8.0	7.1 (15.0)
B-3-1b	57.5	398.0	22.8	52.8	23.7	3.9	3.3 (7.5)

V. CONCLUSION

Equations were obtained which allow to define the bearing capacity of steel-concrete plates during their destruction over the cross-section and along the contact of sheet reinforcement with concrete. It is recommended to find the required intensity of sheet reinforcement anchoring from the condition of equality of ultimate loads.

It is shown that the bearing capacity of reinforced-concrete plates can be significantly increased through substitution of rod-anchoring reinforcement with sheet one, steel consumption being the same.

The analysis of the results of comparison of theoretical data with experimental ones showed that the dependences obtained can be recommended for practical calculations of plates and beams with external reinforcement.

The dependences presented are recommended for calculations of steel-concrete plates for a wide range of concrete (B 15...B50), with steel plate thickness not exceeding 1/40...1/100 of working height of cross-section. While adopting diameter of anchoring rods and steel plate thickness one should meet the requirements of items P 22-391-7 and 8 of Eurocode 4.

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